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LETTER TO THE EDITOR

Critical dynamics of the one-dimensional generalised kinetic Ising model

Ya'akov Achiam†

Nuclear Research Centre—Negev, PO Box 9001, Beer-Sheva and Department of Physics and Astronomy, Tel-Aviv University, Israel

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Abstract. A time-dependent real-space renormalisation group is used to show that all the one-dimensional kinetic Ising models with transition rate that depends on nearest neighbours only, belong to the same universality class of the critical dynamics.

In spite of the recent development in the understanding of critical dynamics, the dynamic scaling hypothesis still has the status of a hypothesis. Therefore it is important to show the existence of universality classes. In this Letter we would like to present a universality class, the kinetic Ising model. This is possible due to the renormalisation group (RG) technique which enables us to gain information on the kinetic behaviour of the system without actually solving the model. The kinetic Ising model (KIM) (Glauber 1963) is the simplest and the most commonly studied model for a non-equilibrium critical dynamics. It is also one of the few models for critical dynamics which has an exact solution in some special cases. Since Glauber (1963) gave a solution to the one-dimensional (1D) case, the model has been generalised to higher dimensionalities, and has been studied using conventional methods (Kawasaki 1972 and references therein) and by the modern (RG) technique (Achiam 1978, 1980, Achiam and Kosterlitz 1978, Hohenberg and Halperin 1977).

The kinetics of the model is introduced by a transition rate $W_i(\sigma_i)$ between the two spin states, $\sigma_i = \pm 1$. Glauber (1963) has already pointed out that the transition rate determined by the detailed balance is not unique. Since the model is an empirical one, there is no *a priori* reason to choose one form of W and not another. Two forms of W are usually used. The one originally chosen by Glauber (1963) is

$$W_i(\sigma_i) = (1 - \sigma_i \tanh E_i)/2, \tag{1}$$

where E_i is the reduced field at site i contributed by the nearest neighbour (NN) σ_j, $E_i = K \sum_j \sigma_j$. The second choice, which is commonly used for the real-space RG calculations (Achiam and Kosterlitz 1978), is

$$W_i^{\mathrm{A}}(\sigma_i) = (P_{\mathrm{e}}(-\sigma_i)/P_{\mathrm{e}}(\sigma_i))^{1/2}, \tag{2}$$

where $P_{\mathrm{e}}(-\sigma_i)$ is the equilibrium probability distribution of the set of spins $\{\sigma_j\}$, where σ_i has been substituted by $-\sigma_i$. Recently, J C Kimball (1979, unpublished report,

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Berkeley, California) studied a 1D KIM with a transition rate which does not vanish as the temperature goes to zero. He found a critical slowing-down similar to the one found in Glauber's original model, and a faster transient.

In this Letter we present an RG analysis of a generalised 1D Glauber model. This generalisation, which has already been suggested by Glauber, includes Kimball's W as a special case. There are two reasons for carrying out the following study of the generalised model.

The first one is to study a variety of W's belong to the generalised model. We found for all of them the same critical slowing-down; hence the details of W are irrelevant (in the RG sense), and all these kinetic models belong to one universality class, in agreement with the dynamic scaling hypothesis (Halperin and Hohenberg 1969, Ferrel *et al* 1968). This result is obtained by applying the decimation RG transformation (Nelson and Fisher 1975) which can be performed exactly in 1D.

The second reason is that the RG study of the generalised model, and in particular of Kimball's case which belongs to it, enables us to understand better how transients enter into the time-dependent RG (TRG) analysis.

The 1D KIM assumes that the spin system $\{\sigma_i\}$ relaxes by a series of single spin flips. The equilibrium probability distribution $P_e(\{\sigma_i\})$ is characterised by a reduced Hamiltonian $H = (\Sigma_i \sigma_i E_i)/2$. The master equation which describes the KIM is

$$\tau\, dP(\{\sigma\}, t)/dt = -\sum_i (W(\sigma_i)P(\sigma_i, t) - W_i(-\sigma_i)P(-\sigma_i, t)), \qquad (3)$$

where by $W(-\sigma_i)P(-\sigma_i, t)$ we mean that all the other spins, $j \neq 1$, have the same value as in the LHS of (3), and τ is the relaxation time of a single spin with the heat bath. The following assumptions determine the spin-flip rate W_i: (i) only one spin changes at a time; (ii) the W_i's fulfil the detailed balance; (iii) the W_i's depend only on the NN spins of i; (iv) the W_i's are invariant under a spin reversal. Assumption (i) was used in expressing equation (3) in its present form. Assumption (ii) means that $W_i(\sigma_i)P(\sigma_i)$ is independent of the value of σ_i (Achiam and Kosterlitz 1978). Both equations (1) and (2) are examples of such W_i's. Assumption (iii) states that a whole set of W's is obtained by multiplying a certain form of W_i, say $W_i^{\rm A}$ (2), by a function $f(\sigma_{i-1}, \sigma_{i+1})$. Assumption (iv) restricts f to be of the form

$$f = a + b\sigma_{i-1}\sigma_{i+1}. \qquad (4)$$

The information on the critical behaviour of the KIM is obtained by applying the RG transformation to the master equation. In this Letter we use the simple RG decimation transformation (Nelson and Fisher 1975). That is, we perform a trace over the two values of each second spin in the master equation (3). This procedure is equivalent to scaling the space by a factor $b = 2$. We examine $P(\sigma, t) = P_e(\sigma)\phi(\sigma, t)$, where the perturbation from equilibrium, ϕ, is given in the linear response approximation by

$$\phi(\sigma, t) = 1 + h(t) \sum_j \sigma_j. \qquad (5)$$

This perturbation forms under the TRG transformation an invariant subspace of perturbations (Achiam 1979a).

It is convenient to write the master equation using (2), (5) as

$$\tau\, dP(\sigma, t)/dt = -\sum_i P_e W_i^{\rm A} (a\sigma_i + b\sigma_{i-1}\sigma_i\sigma_{i+1}). \qquad (6)$$

The RG transformation is performed by summing each second spin over its two values.

The RG transformation of the LHS is the usual static decimation (Nelson and Fisher 1975)

$$\text{RG}[P(\sigma, t)] = P'(\mu, t) \qquad (\mu_\alpha = \pm 1), \tag{7}$$

where the prime means that K and h are replaced by K' and h' respectively such that

$$K' = \tanh^{-1}(\tanh^2 K), \qquad h'(t) = \lambda h(t), \quad \lambda = 2. \tag{8}$$

The RG transformation of each term in the RHS of (5) is performed exactly as for the LHS. The transformation can be easily performed when one recognises that $P_e W_i$ is nothing other than P_e in which the interactions around σ_i have been put to zero (Achiam and Kosterlitz 1978). The transformation of similar expressions (in a different context) has been done elsewhere (Achiam 1979b). We quote here the results:

$$\text{RG}[P_e W_i^{\text{A}}(a\sigma_i + b\sigma_{i-1}\sigma_i\sigma_{i+1})] = -\sum_\alpha P_e'(\mu_\alpha) W_\alpha^{\text{A}}{}'(\mu_\alpha)(a'\mu_\alpha + b'\mu_{\alpha-1}\mu_\alpha\mu_{\alpha+1}),$$

where

$$\begin{pmatrix} a \\ b \end{pmatrix}' = \boldsymbol{\Omega}\begin{pmatrix} a \\ b \end{pmatrix}, \qquad \boldsymbol{\Omega} = \begin{pmatrix} \cosh K/\cosh 2K & 0 \\ 0 & \sinh K/\cosh 2K \end{pmatrix}. \tag{9}$$

The TRG transformation is terminated by scaling the timescale τ:

$$\tau' = b^z \tau, \qquad b^z = \lambda/\omega, \tag{10}$$

where ω is the largest eigenvalue of $\boldsymbol{\Omega}$. This scaling will leave the master equation invariant under the TRG. Standard RG arguments associate z with the dynamic critical exponent (Hohenberg and Halperin 1977).

The term in W associated with a is the one contributing the slow mode; the term with the b is responsible for the transient. However, as we approach the fixed point of the RG transformation, $K^* = \infty$, the two eigenvalues of $\boldsymbol{\Omega}$ become identical. Only one slow timescale with $z = 2$ characterises the system. The above description is changed when more spin operators are included in ϕ. In this case there are contributions to the term linear in σ from higher-order terms, while after the renormalisation, the linear term contributes only to itself. Hence the slowest timescale, associated with the largest eigenvalue of $\boldsymbol{\Omega}$, is the same as before. The fast transients will be fast even at the fixed point.

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